ECEN 314: Signals and Systems
Homework 2

- Date Assigned: September 10, 2010
- Date Due: September 16, 2010

I Problems

1. A continuous time signal $x(t)$ is shown in Fig 1.Sketch and label carefully each of the following signals:

   (a) $x(4 - \frac{1}{2})$
   (b) $[x(t) + x(-t)]u(t)$
   (c) $x(t)[\delta(t + \frac{3}{2}) - \delta(t - \frac{3}{2})]$

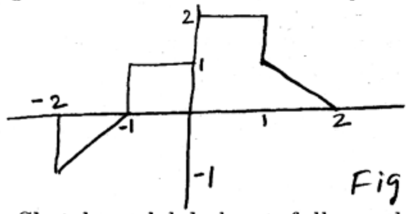

Fig 1.

2. A discrete-time signal is shown in Fig 2. Sketch and label carefully each of the following signals:

   (a) $x[n]u[3 - n]$
   (b) $x[n - 2]\delta[n - 2]$
   (c) $x[(n - 1)^2]$

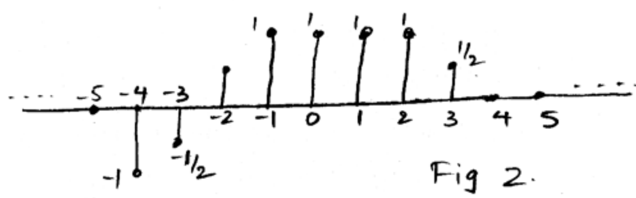

Fig 2.

3. Determine whether or not each of the following continuous time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x(t) = e^{j(\pi t - 1)}$
   (b) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$
   (c) $x(t) = Even\{\cos(4\pi t)u(t)\}$
   (d) $x(t) = Even\{\sin(4\pi t)u(t)\}$
   (e) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)u(2t-n)}$

4. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x[n] = \cos(\frac{\pi}{8}n^2)$
   (b) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

5. Determine for each of the continuous time signals if the following properties hold or do not hold. Memoryless, time invariant, linear, causal, stable. Justify your answers. $y(t)$ is the system output and $x(t)$ is the system input.

   (a) $y(t) = x(t - 2) + x(2 - t)$
   (b) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$
   (c) $y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases}$
   (d) $y(t) = \begin{cases} 0 & x(t) < 0 \\ x(t) + x(t-2) & x(t) \geq 0 \end{cases}$

6. Let $x(t)$ be the continuous time complex exponential signal $x(t) = e^{j\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$-that is, $x[n] = x(nT) = e^{j\omega_0 nT}$. Suppose that $x[n]$ is periodic–that is, $T/T_0 = p/q$, where p and q are integers.

   (a) What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

   (b) Determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.

7. Let $x(t)$ and $y(t)$ be two signals; then the correlation function is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)t(\tau)d\tau$$

   The function $\phi_{xx}(t)$ is usually referred to as the autocorrelation function of the signal $x(t)$, while $\phi_{xy}(t)$ is often called a cross-correlation function.

   (a) What is the relationship between $\phi_{xx}(t)$ and $\phi_{xy}(t)$?

   (b) Compute the odd part of $\phi_{xx}(t)$.

   (c) Suppose that $y(t) = x(t+T)$. Express $\phi_{xy}(t)$ and $\phi_{yy}(t)$ in terms of $\phi_{xx}(t)$.