

2) Let  $x[n]$  be a periodic DT signal with  $N=9$   
 The DTFS coeffs are given by:

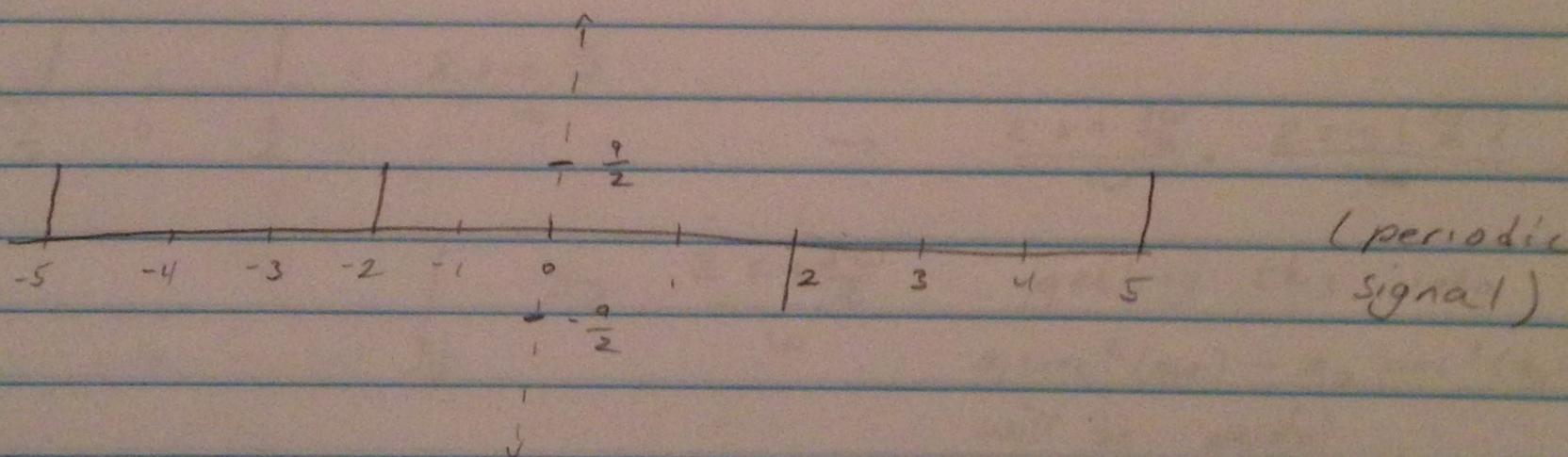
$$x[k] = \cos\left(\frac{10\pi k}{9}\right) + j \sin\left(\frac{4\pi k}{9}\right)$$

Plot the signal  $x[n]$

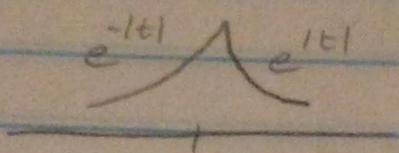
$$\begin{aligned} x[k] &= \frac{1}{2} e^{-j\frac{10\pi k}{9}} - \frac{1}{2} e^{-j\frac{4\pi k}{9}} + \frac{1}{2} e^{j\frac{4\pi k}{9}} + \frac{1}{2} e^{j\frac{10\pi k}{9}} \\ &= \frac{1}{2} e^{-j5\omega_0 k} - \frac{1}{2} e^{-j2\omega_0 k} + \frac{1}{2} e^{j2\omega_0 k} + \frac{1}{2} e^{j5\omega_0 k} \end{aligned}$$

$$x[k] = \frac{1}{N} \sum x[n] e^{-jk\omega_0 n}$$

$$x[5] = \frac{9}{2}, \quad x[2] = -\frac{9}{2}, \quad x[-2] = \frac{9}{2}, \quad x[-5] = \frac{9}{2}$$



1.  $x(t)$  periodic signal with  $T=4$   
 $x(t) = e^{-|t|}$   $0 \leq |t| < 1$   
 $= 0$   $1 \leq |t| < 2$



a) find coeff and show they are real  $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$x[k] = \frac{1}{T} \int_{-1}^1 e^{-|t|} e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-1}^0 e^t e^{-jk\omega_0 t} dt + \frac{1}{T} \int_0^1 e^{-t} e^{-jk\omega_0 t} dt$$

$$\frac{1}{T} \int_{-1}^0 e^{t(1-jk\omega_0)} dt + \frac{1}{T} \int_0^1 e^{-t(1+jk\omega_0)} dt$$

$$= \frac{1}{T} \left[ \frac{e^{t(1-jk\omega_0)}}{1-jk\omega_0} \right]_{-1}^0 + \frac{1}{T} \left[ \frac{e^{-t(1+jk\omega_0)}}{-(1+jk\omega_0)} \right]_0^1$$

$$= \frac{1}{T} \left[ \frac{1 - e^{-1+jk\omega_0}}{1-jk\omega_0} \right] + \frac{1}{T} \left[ \frac{1 - e^{-(1+jk\omega_0)}}{1+jk\omega_0} \right]$$

$x^*[k] \Rightarrow$  the terms will swap

$x[k]$  is real if  $x[k] = x^*[k]$

$$x^*[k] = \frac{1}{T} \left[ \frac{1 - e^{-1-jk\omega_0}}{1+jk\omega_0} \right] + \frac{1}{T} \left[ \frac{1 - e^{-1+jk\omega_0}}{1+jk\omega_0} \right] = x[k]$$

b) what % is DC component of power?

Power of DC  
Total DC

$$\text{DC power} = |x[0]|^2$$

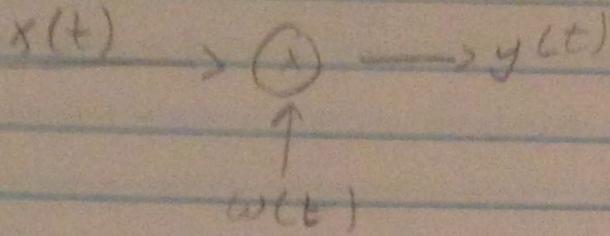
$$x[0] = \frac{1}{4} \left[ \frac{1 - e^{-1}}{1} + \frac{1 - e^{-1}}{1} \right] = \frac{2(1 - e^{-1})}{4} = \frac{1}{2} (1 - e^{-1})$$

$$\text{Total power} = \int_{-1}^0 e^{2t} dt + \int_0^1 e^{-2t} dt = 2 \left[ \frac{e^{-2t}}{-2} \right]_0^1 = 1 - e^{-2}$$

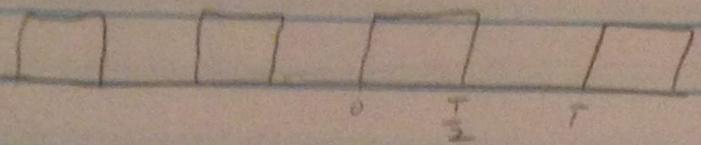
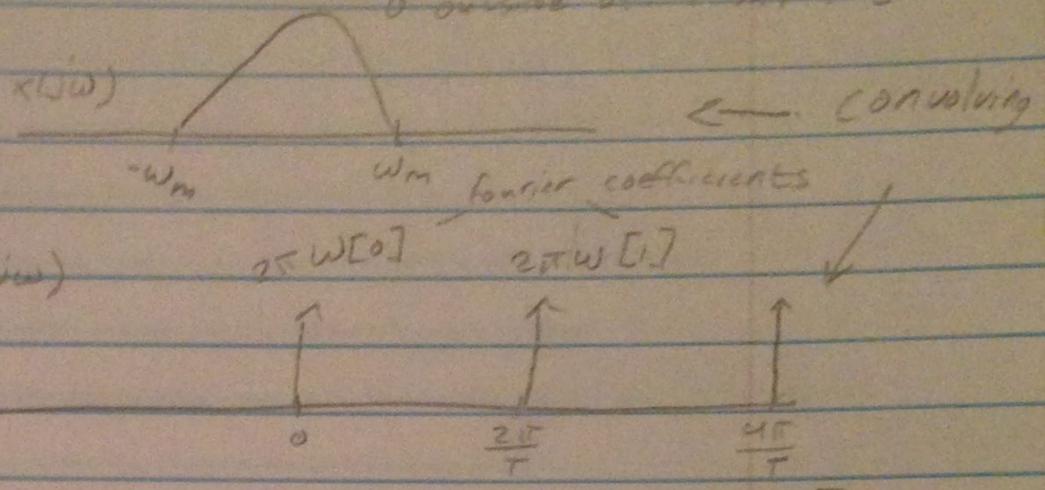
$$\frac{\text{DC Power}}{\text{Total Power}} = \frac{\frac{1}{4} (1 - e^{-1})^2}{(1 - e^{-2})}$$

Final exam

8)

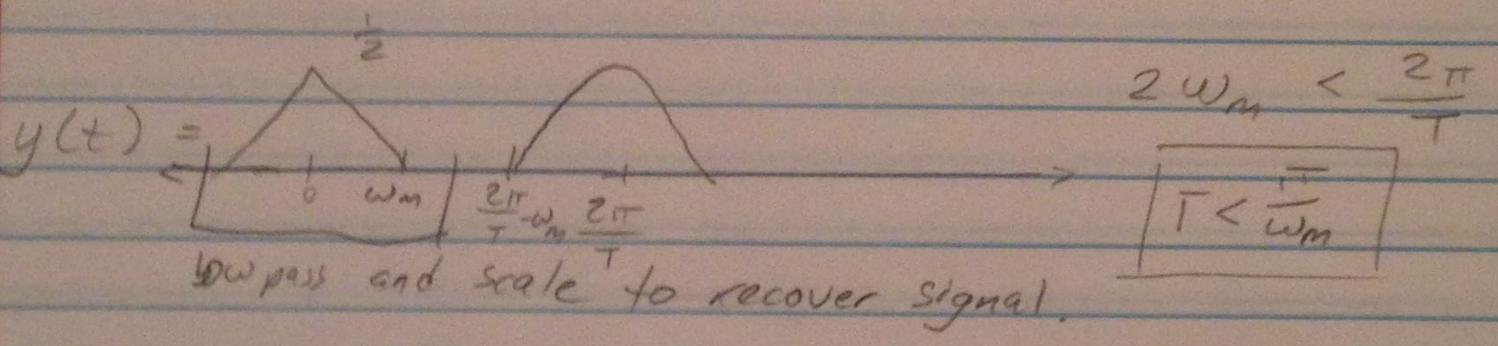


arbitrary signal  
0 outside of  $|\omega_m|$  range



$$W[K] = \frac{1}{T} \int_0^{T/2} e^{-jk\omega_0 t} dt \rightarrow \frac{1}{T} \left[ \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_0^{T/2} = \frac{1}{T} \left[ \frac{1 - e^{-jk\omega_0 (T/2)}}{jk\omega_0} \right]$$

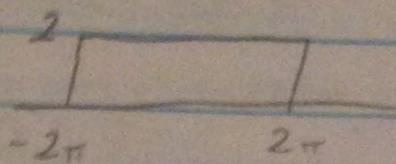
$W[0] = 1, W[1] = \dots$ , all non zero



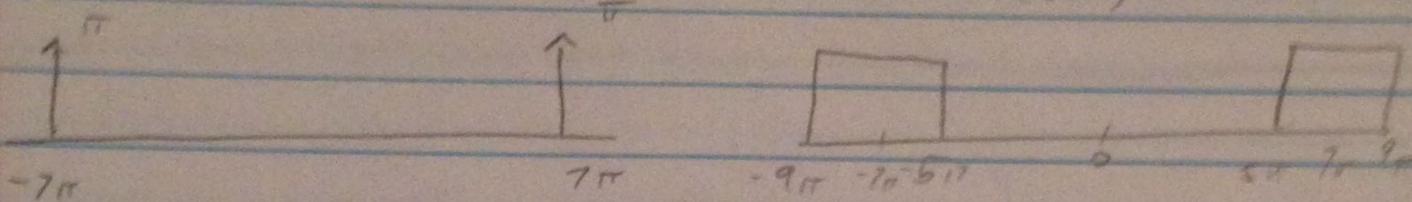
impulse response will be  $\frac{2 \sin(\omega_m t)}{\pi t}$  (low pass filter)

4)  $h(t) = 2 \frac{\sin(2\pi t)}{\pi t} \cos(7\pi t)$

$H(j\omega) = \frac{1}{2\pi} [ \quad ] * [ \quad ]$



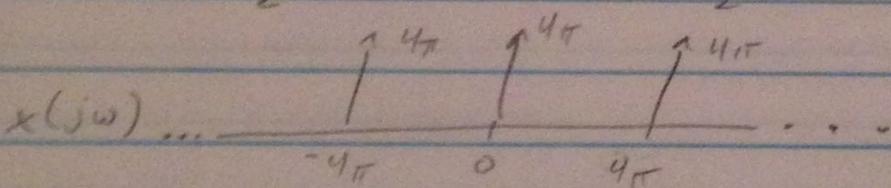
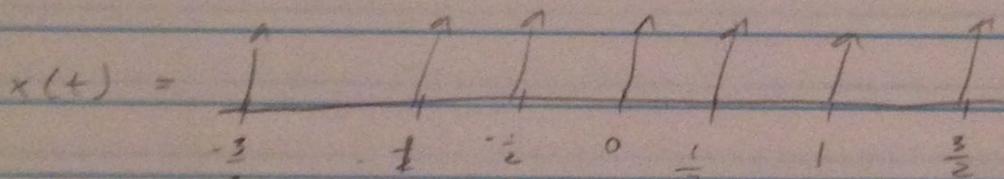
$H(j\omega)$



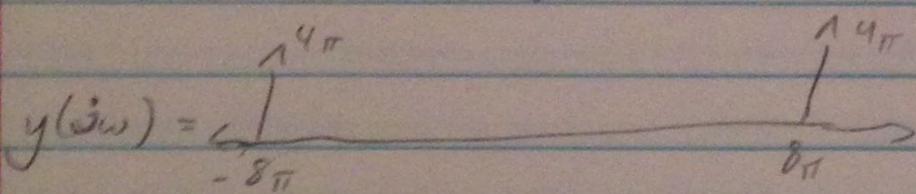
transfer function

compute output when input

$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{1}{2}n) \quad T = \frac{1}{2}$

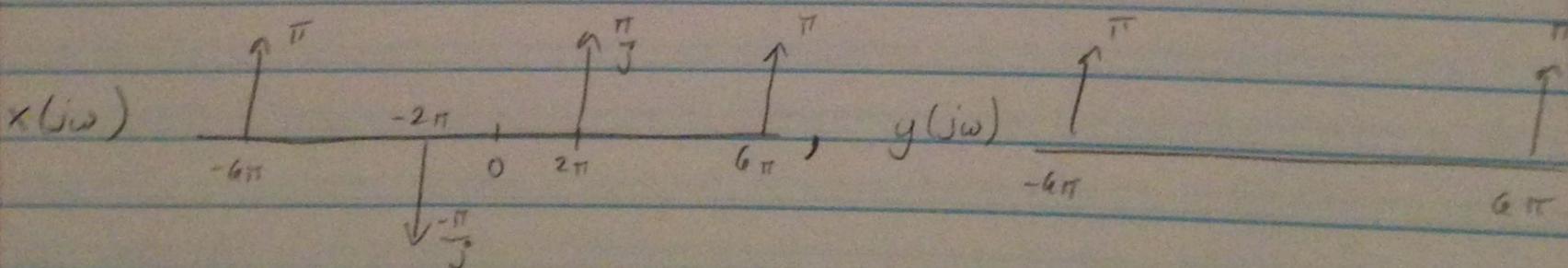


$x(j\omega) \cdot H(j\omega)$

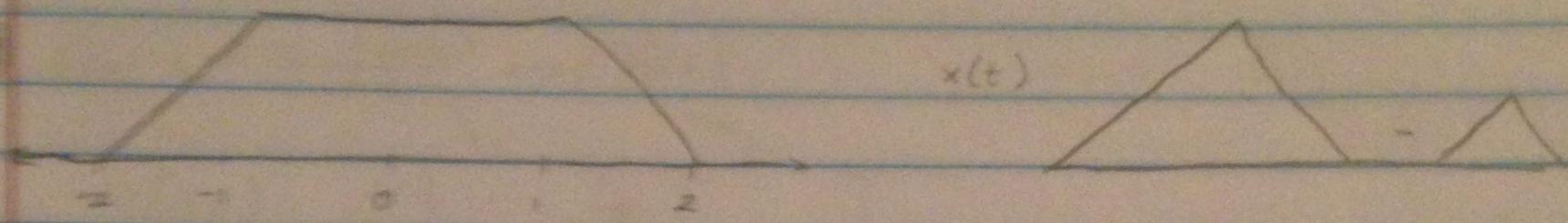


$y(t) = 4 \cos(8\pi t)$

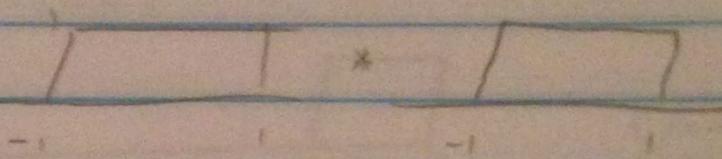
$x(t) = \sin(2\pi t) + \cos(6\pi t)$



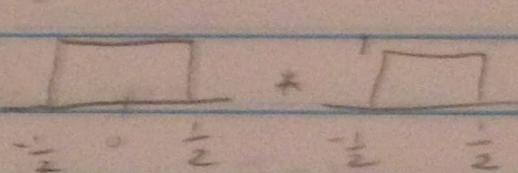
$y(t) = \cos(6\pi t)$



$$x(\omega) = a_1 \text{sinc}^2(b_1 \omega) - a_2 \text{sinc}^2(b_2 \omega)$$

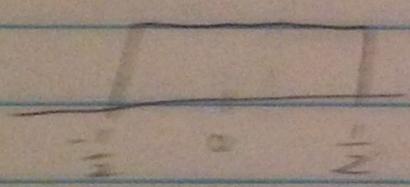
big triangle is convolution of 

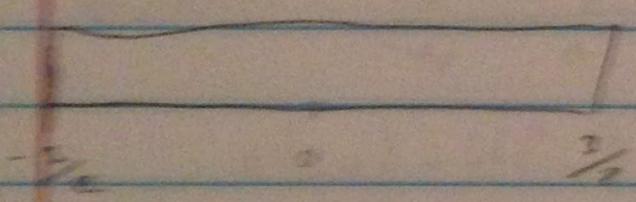
$$x(\omega) = \left( \frac{2 \sin \omega}{\omega} \right)^2 = 4 \text{sinc}^2 \left( \frac{\omega}{\pi} \right)$$

small triangle =  =  $\left( \frac{2 \sin \frac{\omega}{2}}{2 \cdot \frac{\omega}{2}} \right)^2 = \text{sinc}^2 \left( \frac{\omega}{2\pi} \right)$

another way:

convolution of two rectangles of differing width

  $\frac{2 \sin \frac{\omega}{2}}{\omega}$   $\rightarrow$   $\frac{2 \sin \frac{3\omega}{2}}{\omega} \cdot \frac{2 \sin \left( \frac{\omega}{2} \right)}{\omega}$

  $\Rightarrow \frac{2 \sin \frac{3\omega}{2}}{\omega}$  getting this into  $a_1 \text{sinc}^2(b_1 \omega) - a_2 \text{sinc}^2(b_2 \omega)$  will be painful

## Problem 5

$$Y(j\omega) (1+j\omega) = X(j\omega) (3+j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3+j\omega}{1+j\omega}$$
$$= 1 + \frac{2}{1+j\omega}$$

Impulse response

$$h(t) = \delta(t) + 2e^{-t} u(t)$$

$$\text{when } x(t) = e^{-t} u(t) \quad x(j\omega) = \frac{1}{1+j\omega}$$

$$Y(j\omega) = \frac{3+j\omega}{(1+j\omega)^2} = \frac{1}{1+j\omega} + \frac{2}{(1+j\omega)^2}$$

$$y(t) = e^{-t} u(t) + 2te^{-t} u(t)$$

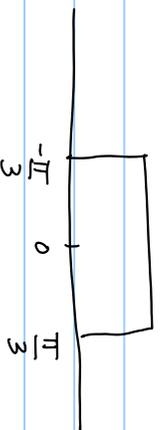
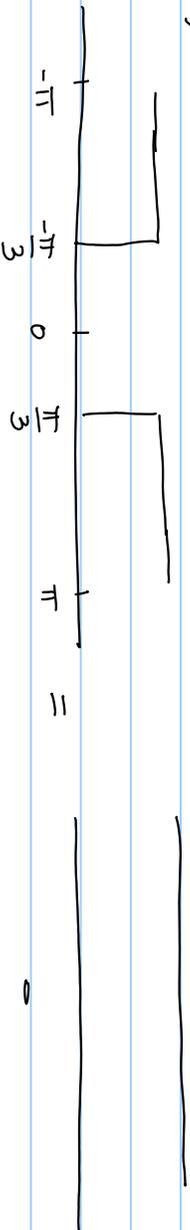
c) Notice that  $Y(j\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$

$$Y(j0) = \int_{-\infty}^{\infty} y(t) dt$$

$$\therefore \int_{-\infty}^{\infty} y(t) dt = Y(j0) = \frac{3+j0}{1+j0} = 3$$

6)

$H(e^{j\omega})$



$$h[n] = \delta[n] - \frac{\sin \frac{\pi}{3} n}{\pi n} = \delta[3] - \frac{1}{3} \operatorname{sinc}\left(\frac{n}{3}\right)$$

$h[n] = 0$  for  $n = 3k$ ,  $k$  is an integer