ECEN 314: Signals and Systems - Homework 4

- Date Assigned: September 26, 2017
- Date Due: October 02, 2017

I Reading Exercise

Sections 1.5 and 1.6

II Problems to turn in

1. OWY 1.20
2. OWY 1.27 a, d, f
3. OWY 1.28 a, b, c
4. OWY 1.31

Solution:

1.16

a

The system is not memoryless because $y[n]$ depends on past values of $x[n]$

b

The output of the system will be $y[n] = \delta[n]\delta[n-2] = 0$

c

From the result of part (b), we may conclude that the system output is always zero for inputs of the form $\delta[n-k], k \in \mathcal{I}$. Therefore, the system is not invertible

1.19

a

Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \to y_1(t) = t^2 x_1(t-1)$$

$$x_2(t) \to y_2(t) = t^2 x_2(t-1)$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = a x_1(t) + b x_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= t^2 x_3(t-1) \\ &= t^2 \left(a x_1(t-1) + b x_2(t-1)\right) \\ &= a y_1(t) + b y_2(t) \end{aligned}$$

b

Consider an arbitrary input $x_1(t)$. Let

$$y_1(t) = t^2 x_1(t-1)$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t - t_0)$$

The output corresponding to this input is

$$y_2(t) = t^2 x_2(t-1) = t^2 x_1(t - 1 - t_0)$$

Also note that

$$y_1(t - t_0) = (t - t_0)^2 x_1(t - 1 - t0) \neq y_2(t)$$

Therefore, the system is not time-invariant

1.20

a

Given

$$x(t) = e^{j2t} \rightarrow y(t) = e^{j3t}$$

$$x(t) = e^{-j2t} \rightarrow y(t) = e^{-j3t}$$

Since the system is linear,

$$x(t) = \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) \rightarrow y_1(t) = \frac{1}{2}\left(e^{j3t} + e^{-j3t}\right)$$

Therefore,

$$x_1(t) = \cos(2t) \rightarrow y1(t) = \cos(3t)$$

b

We know that

$$x_2(t) = \cos\left(2\left(t - \frac{1}{2}\right)\right) = \frac{e^{-j}e^{j2t} + e^{j}e^{-j2t}}{2}$$

Using the linearity property, we may once again write

$$x_1(t) = \frac{1}{2}(e^{-j}e^{j2t} + e^{j}e^{-j2t}) \to y_1(t)\frac{1}{2}(e^{-j}e^{j3t} + e^{j}e^{-j3t}) = \cos(3t - 1)$$

Therefore,

$$x_1(t) = \cos(2(t - 1/2)) \to y_1(t) = \cos(3t - 1)$$

1.27

a

Linear, stable

d

Linear, causal, stable

f

Linear, stable

1.28

a

Linear, stable

b

Time invariant, linear, causal, stable

c

Memoryless, linear, causal

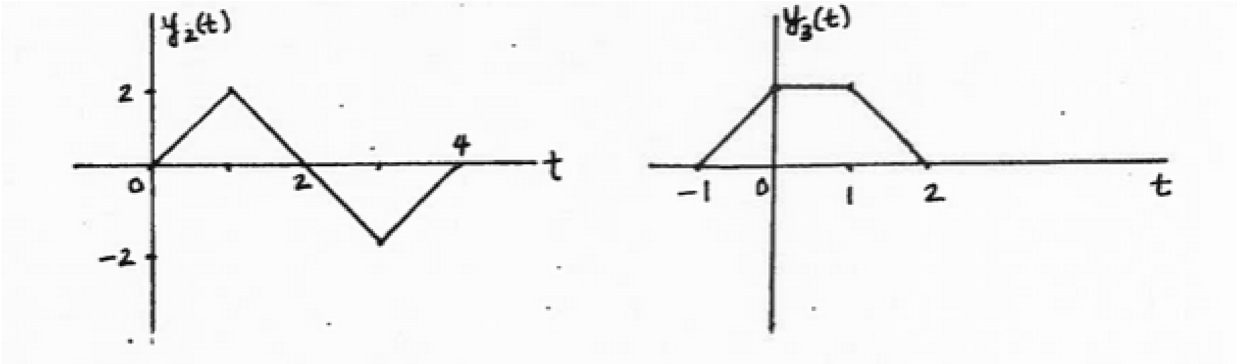
1.31

a

Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using the linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is shown below.

b

Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using the linearity we get $y_3(t) = y_2(t) + y_2(t+1)$. This is shown below.



III Problems to turn in for honors students

1. OWY 1.42
2. OWY 1.43

Solution:

1.42

a

Consider two systems S_1 and S_2 connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to S_1, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to S_2, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since S_1 is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t)$$

where a and b are constants. Since S_2 is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t)$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,\ S_2} az_1(t) + bz_2(t)$$

Therefore, the series combination of S_1 and $S_2$is linear.

Since S_1 is time invariante, we may write

$$x_1(t - T_0) \xrightarrow{S_1} y_1(t - T_0)$$

and

$$y_1(t-T_0) \overset{S_2}{\rightarrow} z_1(t-T_0)$$

Therefore,

$$x_1(t-T_0) \overset{S_1,S_2}{\rightarrow} z_1(t-T_0)$$

Therefore, the series combination of S_1 and $S_2 Eistimeinvariant$

b

False. Let $y(t) = x(t) + 1$ and $z(t) = y(t) - 1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

c

Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$y[n] = z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2]$$

$$= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

The overall system is linear and time-invariant

1.43

a

We have

$$x(t) \overset{S}{\rightarrow} y(t)$$

Since S is time-invariant,

$$x(t-T) \overset{S}{\rightarrow} y(t-T)$$

Now, if $x(t)$ is periodic with period T, $x(t) = x(t-T)$. Therefore, we may conclude that $y(t) - y(t-T)$. This implies that $y(t)$ is also periodic with period T. A similar argument may be made in discrete time.